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# Comments on the Analysis of Total Photoabsorption Measurements in the Energy Range 10 - 150 MeV

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**U.S. DEPARTMENT OF COMMERCE, Juanita M. Kreps, *Secretary***

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**NATIONAL BUREAU OF STANDARDS, Ernest Ambler, *Director***



## Abstract

This note deals with details of the procedure used to extract photonuclear cross sections from total photon absorption measurements. We examine closely some of the approximations implicit in the expressions available for the purely electromagnetic cross sections, most especially pair production, triplet production and the Compton effect, which must be subtracted from the measured photoabsorption in order to obtain photonuclear cross sections. We single out those aspects of the expressions for these electromagnetic processes which most warrant further theoretical research in that they constitute the principal source of uncertainty in the extraction of nuclear data.

Key words: Atomic screening corrections; Coulomb corrections; pair production; photonuclear data; total photon absorption cross section; triplet production.



The recent report of Gimm and Hubbell [1] presents details of the procedure used to extract photonuclear cross sections from the total photon absorption measurements made at the MPI in Mainz by Ahrens et al [2]. This note is intended as a supplement to that report. It is our primary intention here to examine more closely some of the approximations implicit in the expressions available for the purely electromagnetic cross sections, most especially pair production, triplet production and the Compton effect, which must be subtracted from the measured photoabsorption in order to obtain photonuclear cross sections. Furthermore, we wish thereby to single out those aspects of the expressions for these electromagnetic processes which most warrant further theoretical research in that they constitute the principal source of uncertainty in the extraction of nuclear data.

We first present a brief resume of the salient aspects of the processes entering these measurements. The total cross section for photon absorption has been measured in a series of elements ranging from Li to Pb, in the energy range 10 MeV to 150 MeV. The results for light elements (Li to Ca) are presented in [2]. More recent, as yet unpublished, measurements have been made for heavier elements, viz., Cu, Sn, Ta and Pb. (In the case of Li and Be, the measurements go to 220 MeV, and for Pb to 160 MeV, but the bulk of the measurements are in the range 10-150 MeV.) Although the primary purpose of these measurements is to obtain photonuclear cross sections, they constitute at the most only 5% of the total cross section (this in the region of the giant dipole resonance, where they are largest: -- for other energies they are even smaller). Thus, depending on the particular energy, between 95 and 100% of the total photoabsorption cross

section is due to atomic processes: Compton effect, pair production (in the field of the nucleus), triplet production (pair production in the field of the atomic electrons) and photoelectric effect. The cross sections for these electromagnetic processes, which are subtracted from the total measured cross section to obtain the photonuclear cross section, must, therefore, be known with great accuracy if useful nuclear data are to be obtained. In those regions where the photonuclear cross section is zero or negligible, these measurements constitute above all a very accurate determination of the atomic cross sections. We will return to this point in our final comment at the end of this note. This is the case, for example, for heavy elements at high energies ( $Z \geq 50$  and 100 - 150 MeV), and for all elements below particle threshold. This then suggests a number of questions, which are the starting point for these comments.

1) For those regions in which the photonuclear cross section is not negligible, what are the errors in the available expressions for the atomic cross sections, and what limits do these place on the accuracy to which the nuclear cross sections may be known.

2) In those regions in which the photonuclear cross sections are negligible, does the accuracy of the present measurements permit one to clarify or resolve any of the uncertainties in the theoretical expressions for the cross sections for the atomic processes.

3) Are there, currently available, theoretical expressions for the atomic cross sections of greater accuracy than those employed in the analysis described in [1].

4) In what direction do these absorption measurements suggest further research that would improve upon the currently available theoretical expressions for the atomic cross sections.

We start by presenting some figures and tables that contain the essentials for our comments. In figs. 1-4 we show curves of the three major atomic processes occurring here: pair production, Compton effect, and triplet production, as well as the total atomic cross section, as a function of the photon energy, for four elements: Li, Ca, Sn, and Pb ( $Z = 3, 20, 50, \text{ and } 82$ ). Table I gives, for these same elements, at each of three different energies, the cross sections for each of these three atomic processes as percentages of the total absorption cross section ( $\sigma_{\text{tot}}$ ). Table II gives, for the same elements and energies as in Table I, the screening and Coulomb corrections, again as percentages of  $\sigma_{\text{tot}}$ . The photoelectric cross section has been included in the actual analysis, but is not shown in these figures and tables since it is always less than 1% of  $\sigma_{\text{tot}}$ .

Next we note the essential formulae used in the analysis:

$$\sigma_{\text{nucl}} = \sigma_{\text{tot}} - (\sigma_{\text{ph}} + \sigma_{\text{c}} + \sigma_{\text{t}} + \sigma_{\text{p}}) \quad (1)$$

$$\sigma_{\text{p}} = (\sigma_{\text{BH}} - \Delta S - \Delta C) \cdot f_{\text{rad}} \quad (2)$$

Here  $\sigma_{\text{nucl}}$  = photonuclear cross section  
 $\sigma_{\text{tot}}$  = total measured photoabsorption cross section  
 $\sigma_{\text{ph}}, \sigma_{\text{c}}, \sigma_{\text{t}}, \sigma_{\text{p}}$  are the cross sections for the atomic processes:  
photoelectric, Compton, triplet, pair production,  
respectively.  
 $\sigma_{\text{BH}}, \Delta S, \Delta C$  and  $f_{\text{rad}}$  all refer to the pair production cross  
section,  $\sigma_{\text{p}}$ :

$\sigma_{\text{BH}}$  is the Born approximation (Bethe-Heitler) cross section for an unscreened (point Coulomb) potential.

$\Delta S$  is the Born approximation screening correction,

$$(\Delta S = \sigma_{\text{screened}} - \sigma_{\text{unscreened}}, \text{ both in Born approximation.})$$

$\Delta C$  is the Coulomb correction in the absence of screening,

$$(\Delta C = \sigma_{\text{with Coulomb corrections}} - \sigma_{\text{BH}}, \text{ both for an unscreened, point Coulomb potential}).$$

$f_{\text{rad}}$  is the radiative correction to the pair production cross section.

As is clear from the figures and tables, the screening and Coulomb corrections,  $\Delta S$  and  $\Delta C$ , are of particular significance for heavy elements ( $Z \gtrsim 50$ ), and for these the absorption is dominated by the pair production cross section. Thus, if we wish to use these measurements as a check on the theoretical expressions for the screening and Coulomb corrections, we must consider heavy elements. Put otherwise: We don't learn anything new about the pair production cross section from measurements on the light elements ( $Z \lesssim 20$ ). For these, the Coulomb correction is very small and the screening correction may be calculated to sufficient accuracy using form factors derived from non-relativistic charge distributions since the difference between the screening correction obtained from relativistic and non-relativistic charge distributions is within the experimental error of these measurements ( $\sim 0.1\%$ ) for light elements.

Next, we note from the expressions (1) and (2) used to obtain the photonuclear cross sections that the screening and Coulomb corrections appear together as  $\Delta S + \Delta C$ . Thus, for a given element and at a given energy, we cannot distinguish, from the measurements, errors in the

screening correction from those in the Coulomb correction. Although in principal these corrections might be separated experimentally by a series of measurements since they differ both in their energy dependence and in their  $Z$  dependence, a much greater experimental accuracy would be required for such a separation. Moreover, such a separation, in which these two corrections are simply added, is itself an approximation which is not fully justified at the level of accuracy aspired to in these measurements. We will comment on this point later in greater detail.

We now examine in some detail a number of the approximations implicit in the theoretical expressions used for the atomic processes of importance here, and consider first the pair production cross section,  $\sigma_p$ , as given by (1).

1) The expression used for the radiative correction to the total cross section for pair production,  $f_{\text{rad}}$ , is that given by Mork and Olsen [3], which is the best available calculation. However, Mork and Olsen only derive the radiative correction to the first Born approximation cross section; the question of the radiative correction to the higher Born terms (the Coulomb correction) is not examined. Since the radiative correction to the first Born approximation cross section is of the order of 1% and the Coulomb correction (e.g., for Pb) is of the order of 10%, the term  $\Delta C \cdot f_{\text{rad}} \sim 0.1\%$ . Thus if this term is wrong by a factor 2, this would introduce an error of the order of 0.1%.

2) Again referring to the radiative correction to pair production, we note that the calculation of Mork and Olsen uses the Weizsäcker-Williams method, which is, in general, valid for very high energies,  $^* k$ .

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\* Throughout, unless otherwise indicated, energies are given in units of the electron rest mass,  $mc^2$ , and momentum in units of  $mc$ .

In particular, it generally neglects terms of order unity relative to those of order  $\ln k$ . One can therefore ask what errors are made by applying it in the energy range of interest here: 10 - 150 MeV. If the radiative correction given by this method is itself off by 10 - 20% at the lower energies, which is not unreasonable, then this would again introduce errors of 0.1 - 0.2% in the total pair cross section.

3) As we have already noted, the screening and Coulomb corrections,  $\Delta S$  and  $\Delta C$  are, in the present analysis, [1], calculated independently and added to the unscreened Born approximation cross section. The rationale for this procedure was given sometime ago by Davies, Bethe and Maximon [4], viz., that for very high energies the Coulomb correction to the total pair cross section comes from momentum transfers  $q \sim 1$ , whereas the screening correction comes from relatively small momentum transfers,  $q \sim \frac{Z^{1/3}}{121}$ . Moreover, an estimate of the error committed by this separation was also given there -- it was not then of significance, but for the present measurements it is no longer totally negligible. From p. 791, Eq. (41) of [4] we have the estimate of the error committed by neglecting the Coulomb corrections for momentum transfers  $q \leq \beta \equiv \frac{Z^{1/3}}{121}$ , viz.,

$$a^2 \cdot \beta^2 (\ln \beta^2)^2 \approx 0.2\% \text{ of } \sigma_p \text{ for Pb, } (a = Z/137). \quad (3)$$

This should only be regarded as an estimate, however. A proper calculation would contain the higher order terms in a screened Coulomb potential directly. Such an approach was given by Olsen, Maximon and Wergeland [5]. There, however, the two significant regions of momentum transfer,  $q \sim 1$  and  $q \sim \frac{1}{E}$ , were separated, and wavefunctions and matrix elements appropriate to each of these regions individually were calculated. The problem

of the overlap of Coulomb and screening corrections did not therefore arise: In the region  $q \sim \frac{1}{E}$  the Coulomb corrections did not enter -- the matrix elements were shown to be identical to those calculated in Born approximation in that region. And in the region  $q \sim 1$ , there were no screening corrections -- one could use the wavefunctions and matrix elements calculated for a point Coulomb field. What is required, therefore, if one is to examine the overlap of screening and Coulomb corrections, are wavefunctions and matrix elements valid throughout the range of significant momentum transfers, from  $q \sim \frac{1}{E}$  to  $q \sim 1$ , approximate only in that terms giving contributions of relative order  $\left(\frac{1}{E}\right)^2$  to the final cross section are neglected.

4) The next question concerns the form factors used in calculating the screening correction,  $\Delta S$ . It should be noted that in order to calculate the total pair production cross section, the range of momentum transfers for which the form factor must be known is

$$\frac{4}{k + \sqrt{k^2 - 4}} \leq q \leq k + \sqrt{k^2 - 4} \quad (4)$$

Thus for large photon energies,  $k$ ,

$$\frac{2}{k} \leq q \leq 2k \quad , \quad (4a)$$

i.e., the form factor must be known both for very small and for very large momentum transfers, although, as is well-known, the contribution to the screening correction from  $q \gtrsim 0(1)$  is very small. The most accurate form factors available in the literature for this range of momentum

transfers are those given by I. Øverbø [6], and these have been used for calculating the screening correction in the present analysis. These form factors have been calculated using a combination of the results of the relativistic Hartree-Fock calculations of Liberman, Cromer and Waber [7a] and of Doyle and Turner [7b]. Details of these calculations are given in [6], [7a] and [7b], and need not be dwelt upon here. Two observations are of particular significance for the present analysis, however. The first is that for heavy elements (e.g., Pb) and at high energies ( $\sim 150$  MeV), the difference between the screening correction calculated from these relativistic charge densities and those calculated from non-relativistic charge densities is approximately 1.2% of the total pair cross section (see fig. 5). This is an order of magnitude larger than the experimental errors claimed for these measurements. The question of the errors implicit in the form factors used here is thus a very important one. In this connection we note first the comment in [6] (p. 332) that "for light elements, really accurate charge densities can only be obtained by treating exchange according to the Hartree-Fock scheme," though for these elements non-relativistic wavefunctions should be "fairly accurate." The question thus remains of estimating the errors in the screening correction obtained from the form factors of ref. [6]. Although no "exact" calculation is possible, one can obtain an estimate of these errors by using charge densities more accurate than those of ref. [7], which formed the basis of the form factors given by Øverbø in [6]. A number of fully relativistic Dirac-Hartree-Fock calculations (in which the exchange is also treated relativistically) have appeared within the last few years. Papers relating to two of these should be noted:

a) That of J. B. Mann and J. T. Waber [8], on "Self-Consistent Relativistic Dirac-Hartree-Fock Calculations on Lanthanide Atoms." The codes for calculating the relativistic DHF wave functions used in that paper are available from J. B. Mann<sup>\*</sup> and could be used to calculate form factors and screening corrections.

b) That of J. P. Desclaux [9], on "A Multiconfiguration Relativistic Dirac-Fock Program." This work is similar to that of Mann and Waber in that it is also a self-consistent Dirac-Hartree-Fock calculation. In fact, both calculations are based on the work of I.P. Grant on the "Relativistic Calculation of Atomic Structures," [10]. The calculation of Desclaux is probably superior to that of Mann and Waber, however, in that the latter uses pure jj coupling, which is not true for many atoms, whereas in the work of Desclaux, an average is taken over all jj configurations arising from a single LS configuration. The details of this procedure are given by Desclaux, Mayers and O'Brien [11], and by Desclaux, Moser and Verhaegen [12]. Theirs seems to be the best available procedure for calculating the charge densities. It would clearly be of interest to compare the form factors and screening corrections obtained with the codes used by these authors with those obtained from the calculation of Mann and Waber and with that of Overbø.

5) The next question concerns the Coulomb correction,  $\Delta C$ . For the heavier elements, it is this correction which introduces the greatest uncertainty in the photonuclear cross sections obtained from these measurements in the manner described here. Indeed, it was already noted in [2]

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\* Joseph B. Mann, LASL, P.O. Box 1663, Los Alamos, N.M. 87545 (private communication)

that the "lack of reliable theoretical calculations for the Coulomb correction between 10 and 50 MeV restricts the range in  $Z$  available for absolute  $\sigma_{\gamma,T}$  measurements to  $Z \cong 20$ ." Specifically, there exist accurate calculations of the Coulomb correction (the correction to the unscreened, Born approximation total cross section for pair production in a point Coulomb field) in two very separated energy regions. For low photon energies -- from threshold ( $2 mc^2$ ) to  $10 mc^2$  (5 MeV), extensive, accurate (having errors of the order of 0.1%) numerical calculations of the Coulomb correction have been given by Øverbø, Mork and Olsen [13]. (For the most detailed account of this calculation one should consult the thesis of I. Øverbø [14].) At the other extreme, for very high energies, the Coulomb correction has been given by Davies, Bethe and Maximon [4]. Unfortunately, there is at present no accurate method of determining the errors in their high-energy-limit expression for the Coulomb correction when applied at the energies of interest for these experiments -- say 10 - 300 MeV. A conservative limit on the error for an incident photon energy of 75 MeV would be, for example for Pb, 10% of the Coulomb correction itself, or 1% of  $\sigma_p$ . This is, however, far from sufficient for the analysis of the present measurements on heavy elements. In order to bridge the gap between the region below  $10 mc^2$  and the high energy limit, an empirical analytic expression given by Øverbø [15], has been used in the present analysis. This expression was derived by representing the Coulomb correction by a simple, reasonable, analytic function of  $Z$  and  $k$  whose form is chosen so that it goes manifestly into the high energy limit for very large  $k$ , and containing a number of arbitrary parameters (which are polynomials in  $Z^2$ ), chosen to fit the known [13], [14] calculated values

in the low energy region,  $3.5 mc^2 \leq k < 10 mc^2$ . With six parameters he is able to fit the values of [13], [14] for all  $Z$  in this low energy region within the errors ( $\sim 0.1\%$  of  $\sigma_p$ ) of the values calculated in [13], [14]. This analytic expression is then assumed to give the Coulomb correction for all higher energies ( $k > 5$  MeV) by extrapolation. While this is not an unreasonable procedure, given the lack of any calculation of sufficient accuracy for heavy elements in the intermediate energy region, 10-150 MeV, it does not enable one to make any statement about the errors that should be associated with these extrapolated values in this intermediate energy region. To elucidate this point we have repeated Overbø's procedure, but have chosen an alternate, equally reasonable analytic function of  $Z$  and  $k$  to represent the Coulomb correction, also of a form such that it goes manifestly into the high energy limit for very large  $k$ , containing eight arbitrary parameters. Specifically, our alternate analytic form has been chosen to have the same energy dependence as that of the first few terms of the high energy expansion of the Born approximation cross section for pair production [16], viz.,

$$\sigma = \sigma_{\text{BH}} + \Delta C$$

$$\begin{aligned}
 = \bar{\phi} \left\{ \left[ \left( \frac{28}{9} \ln 2k - \frac{218}{27} \right) \right. \right. \\
 + \left( \frac{2}{k} \right)^2 \left( 6 \ln 2k - \frac{7}{2} + \frac{2}{3} \ln^3 2k - \ln^2 2k - \frac{1}{3} \pi^2 \ln 2k + 2\zeta(3) + \frac{\pi^2}{6} \right) \\
 - \left( \frac{2}{k} \right)^4 \left( \frac{3}{16} \ln 2k + \frac{1}{8} \right) \\
 \left. \left. - \left( \frac{2}{k} \right)^6 \left( \frac{29}{9 \cdot 256} \ln 2k - \frac{77}{27 \cdot 512} \right) \right] \right\}
 \end{aligned}$$

(5)

$$\begin{aligned}
 + a^2 \left[ C_0 \right. \\
 + \left( \frac{2}{k} \right)^2 \left( C_1 \ln^3 2k + C_2 \ln^2 2k + C_3 \ln 2k + C_4 \right) \\
 + \left( \frac{2}{k} \right)^4 \left( C_5 \ln 2k + C_6 \right) \\
 \left. \left. + \left( \frac{2}{k} \right)^6 \left( C_7 \ln 2k + C_8 \right) \right] \right\}
 \end{aligned}$$

$$\bar{\phi} = \frac{e^2}{\hbar c} \left( \frac{Ze^2}{mc^2} \right)^2 ; \quad \zeta(3) = 1.2020569\dots$$

The absence of a term of order  $\ln 2k$  in the Coulomb correction is perhaps worth noting. It may be understood from the observation that this term comes from the region of very small momentum transfers,  $q \sim q_{\min} \cong \frac{2}{k}$ . But for these momentum transfers the Born approximation is valid, there is no Coulomb correction. Thus the term of order  $\ln 2k$  appears only in the Born approximation. The constant  $C_0$  is given by the high energy limit [4], viz.,

$$a^2 C_0 = - \frac{28}{9} f(Z)$$

$$f(Z) = a^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2+a^2)} \approx a^2 \left[ (1+a^2)^{-1} + 0.20206 - 0.0369a^2 \right. \\ \left. + 0.0083a^4 - 0.002a^6 \right] \quad (6)$$

The parameters  $C_1, \dots, C_8$  are chosen to be second order polynomials in  $a^2 = (Ze^2/\hbar c)^2$ , as in [15]:

$$C_i = \alpha_i + \beta_i a^2 + \gamma_i a^4 \quad (i = 1, 2, \dots, 8) \quad (7)$$

The constants  $\alpha_i$ ,  $\beta_i$  and  $\gamma_i$  have been chosen to give the best possible least squares fit to the analytic expression [15] in the low energy region\*, within which it never differs from that result by more

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\* In fact, we have made a least squares fit to [15] in the somewhat restricted region  $4 \leq k \leq 10 mc^2$ . Since these are, or represent, asymptotic expressions for large  $k$ , they should not be expected to converge well at all for very small  $k$ . Forcing them to do so would, in our opinion, tend to determine constants giving a poorer representation of the function in the intermediate energy region. The maximum difference with the results of [15] just quoted, viz.,  $6 \times 10^{-4} \sigma$ , is for all  $Z \leq 82$  and for the energy region  $4.15 \leq k \leq 10 mc^2$ ,<sup>p</sup> but for most of this region the errors are considerably less.

than  $6 \times 10^{-4} \sigma_p$ . Thus both in the low energy region and at the high energy limit, the two expressions give essentially identical values. The numerical values of the constants  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$  are given in Table III.

The significant difference between the analytic form used here and that of [15] is that in this note the factors of the constants and log terms in  $\Delta C$  are inverse powers of  $k^2$ , as in the Born approximation cross section\* [16], whereas in [15] they are inverse powers of  $k$ . Thus, although the two expressions agree within the low energy region and go to the same high energy limit, they may be expected to give somewhat different results in the intermediate energy region. To show this, is, in fact, the purpose of this exercise. In particular, the expression for  $\Delta C$  presented here will approach the high energy limit more rapidly than that of [15]. It is not that either expression has a particularly greater claim to validity in the intermediate region, but rather that neither can claim to have an uncertainty which is less than the difference of the values given by these two expressions. In fig. 6 we plot the ratio of the difference between the Coulomb correction calculated using our analytical expression and that calculated using the expression in [15] to the Born approximation cross section,  $\sigma_{BH}$ , as a function of photon energy, for Sn and for Pb. Since this difference is essentially zero in the low energy region, on the one hand, and at extremely high energies, on the other, it must either be essentially zero for all intermediate energies or else have a maximum somewhere in the intermediate region. From fig. 6, it is seen that there is a maximum in the neighborhood of 20 MeV, and that the difference there is slightly more than 0.9% of  $\sigma_{BH}$ , or slightly more than 1% of the unscreened pair cross section,  $\sigma_{BH} + \Delta C$ . Since neither expression

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\*It is our belief, though this is at present only a conjecture, that the asymptotic form of the Coulomb correction is also of this form, involving only inverse powers of  $k^2$ .

has, in fact, a basis in theory that would make it more accurate than the other, this implies an uncertainty of at least 1% in the pair cross section at these energies, where, moreover, the photonuclear cross section is largest. The fact that at higher energies, where the nuclear cross section is essentially zero, there is agreement between the measurements and the pair cross section calculated from [15], is not a test of the validity of that expression in the region about 20 MeV: At higher energies ( $\sim 150$  MeV), the asymptotic value has almost been attained, and almost all reasonable expressions will be within a few tenths of a percent of the correct value there. In any event, one observation is eminently clear from fig. 6, namely, that the low energy region,  $1.75 \leq k \leq 5$  MeV, provides far too small a basis for an extrapolation to the region 10-150 MeV. This is all the more true in that the Coulomb correction varies quite rapidly in the low energy region, as may be seen from fig. 7, where the ratio of the Coulomb correction to the Born approximation cross section is plotted as a function of energy. (Values for  $\Delta C$  are from [14] for  $k < 10$   $mc^2$  and from [15] for  $k > 10$   $mc^2$ . Values for  $\sigma_{BH}$  are from [16].) On the other hand, above 10 MeV this ratio varies relatively little. This suggests that an extension of the numerical calculation of  $\sigma_{BH}$  [15] to  $20$   $mc^2$  might well provide a sufficient basis for an extrapolation to higher energies. Such an extension should not be of unsurmountable difficulty, given the improvements in the capabilities of large scale computers that have occurred within the almost ten years since that last calculation.

It is worth noting that this same problem arises in connection with the extraction of nuclear cross sections from the background radiative tail in high energy electron scattering measurements. There it is possible

to determine experimentally the radiative tail in the region near the elastic peak (below particle threshold) and again well beyond the giant resonances, where the scattered electron has radiated most of its energy. Again, a common procedure is to fit the form of the tail in the intermediate region by an empirical analytic expression that fits the experimental radiative tail at the two extremes. In those experiments, the nuclear cross sections are sometimes no more than 5% of the total, so that uncertainties of the order of a percent in the assumed radiative tail in the region of the giant resonances (uncertainties which themselves vary with energy) can contribute large errors in the extracted nuclear cross sections.

6) Our final comments concern the triplet cross section. This is of particular importance for light elements. For example, it is 15% of the total photoabsorption cross section for Li at 100 MeV. Our comments here concern in particular the screening correction as calculated in the present analysis (p. 10, Eq. (37) in [1]). In [1], the recoil momentum distribution for pair production in the field of the nucleus has been used (i.e., the target is assumed to be infinitely heavy). Since the target is, for triplet production, an electron, this procedure is somewhat in error, though only slightly so, since, as was shown by Suh and Bethe [17], the recoil momentum distribution is essentially independent of the target mass for small momentum transfers,  $q \ll 1$ , i.e., for momentum transfers of importance for the screening correction. This error is then corrected (Eq. (39) in [1]) by multiplying the screening correction by a factor,  $f_{ret}$ , taken to be the ratio of the unscreened triplet cross section to the unscreened Bethe-Heitler pair cross section.

A more correct procedure would be simply to use the recoil distribution proper to triplet production. This has been given by Borsellino [18]. His expression neglects only the  $\gamma$ -e and exchange diagrams, whose contribution is only of order 1% of the triplet cross section for the energies of interest here ( $> 10$  MeV). (See Mork [19], and Haug [20], for a discussion of this point.) We would therefore suggest that the screening correction for triplet cross section be calculated using the recoil momentum distribution given by Borsellino, and that the results thus obtained be compared with those obtained in [1].

As mentioned in [1], the aim of the total absorption cross section measurements for high  $Z$  elements is to check the nonnuclear cross sections rather than to perform a precise measurement of the nuclear cross section. For  $Z \geq 50$ , the photonuclear cross section is almost entirely due to  $(\gamma, n)$ ,  $(\gamma, 2n)$ , ... reactions, and the most accurate measurements of these photoneutron cross sections are those performed at Illinois, Livermore, and, most especially, by Bergère et al at Saclay [21]. The present total photoabsorption measurements may thus be used in conjunction with the photoneutron measurements to obtain the best experimental determination of the atomic cross section:

$$\sigma_a = \sigma_{tot} - \left[ \sigma(\gamma, n) + \sigma(\gamma, 2n) + \dots \right], \quad (8)$$

as was in fact suggested by the group at Mainz several years ago [22]. We note further that for heavy elements the pair production cross section dominates the nonnuclear cross section (it is over 65% of  $\sigma_{tot}$  for  $Z \geq 50$  and  $k \geq 10$  MeV). Thus, upon subtracting the relatively small

cross sections for Compton, triplet and photoelectric effect (calculated from theory), we may use the total photoabsorption measurements to obtain the best experimental determination of the pair production cross section as a function of photon energy:

$$\sigma_p = \sigma_{\text{tot}} - \left[ \sigma(\gamma, n) + \sigma(\gamma, 2n) + \dots \right] - \left[ \sigma_c + \sigma_t + \sigma_{\text{ph}} \right] . \quad (9)$$

This may then be compared directly with the best currently available theoretical determination, given recently by Øverbø [23].

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TABLE I

Total Cross Sections for Pair Production, Compton Effect,  
and Triplet Production as Percentages of the Total Absorption  
Cross Section,  $\sigma_{\text{tot}}$ , for Li, Ca, Sn and Pb

		20 MeV	80 MeV	150 MeV
Li	pp	23	53	61
	Compton	71	31	19
	triplet	5.5	16	20
Ca	pp	66	88	91
	Compton	31	8	4.5
	triplet	2.4	4	4.5
Sn	pp	82	94	96
	Compton	16.5	3.9	2.1
	triplet	1.3	1.8	2.0
Pb	pp	87	96	97
	Compton	12	3	1.4
	triplet	.9	1	1.3

TABLE II

Screening and Coulomb Corrections ( $\Delta S$  and  $\Delta C$ ) to the Total Pair Production Cross Section, as Percentages of the Total Absorption Cross Section,  $\sigma_{\text{tot}}$ , for Li, Ca, Sn, and Pb

		20 MeV	80 MeV	150 MeV
Li	$\Delta S$	-	1	4
	$\Delta C$	-	-	-
Ca	$\Delta S$	1.5	8	14
	$\Delta C$	.5	.7	.7
Sn	$\Delta S$	3.6	13	21
	$\Delta C$	3.8	4.4	4.3
Pb	$\Delta S$	5.7	18	27
	$\Delta C$	11	11.5	11.1

TABLE III

Numerical Values of Constants  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$  (Eq (7)),  
Used in Evaluating the Coulomb Correction Given in Eq (5).

$i$	$\alpha_i$	$\beta_i$	$\gamma_i$
1	20.692201	- 12.316591	.9068565
2	- 77.493212	30.661404	3.7151656
3	60.915963	- .74569210	- 7.7968410
4	59.942296	- 17.101498	- 8.4300644
5	- 30.472058	- 37.670672	- 1.8958007
6	- 26.309659	- 6.7556663	7.4799697
7	4.4037133	37.244493	26.205729
8	19.433380	30.092603	18.048061



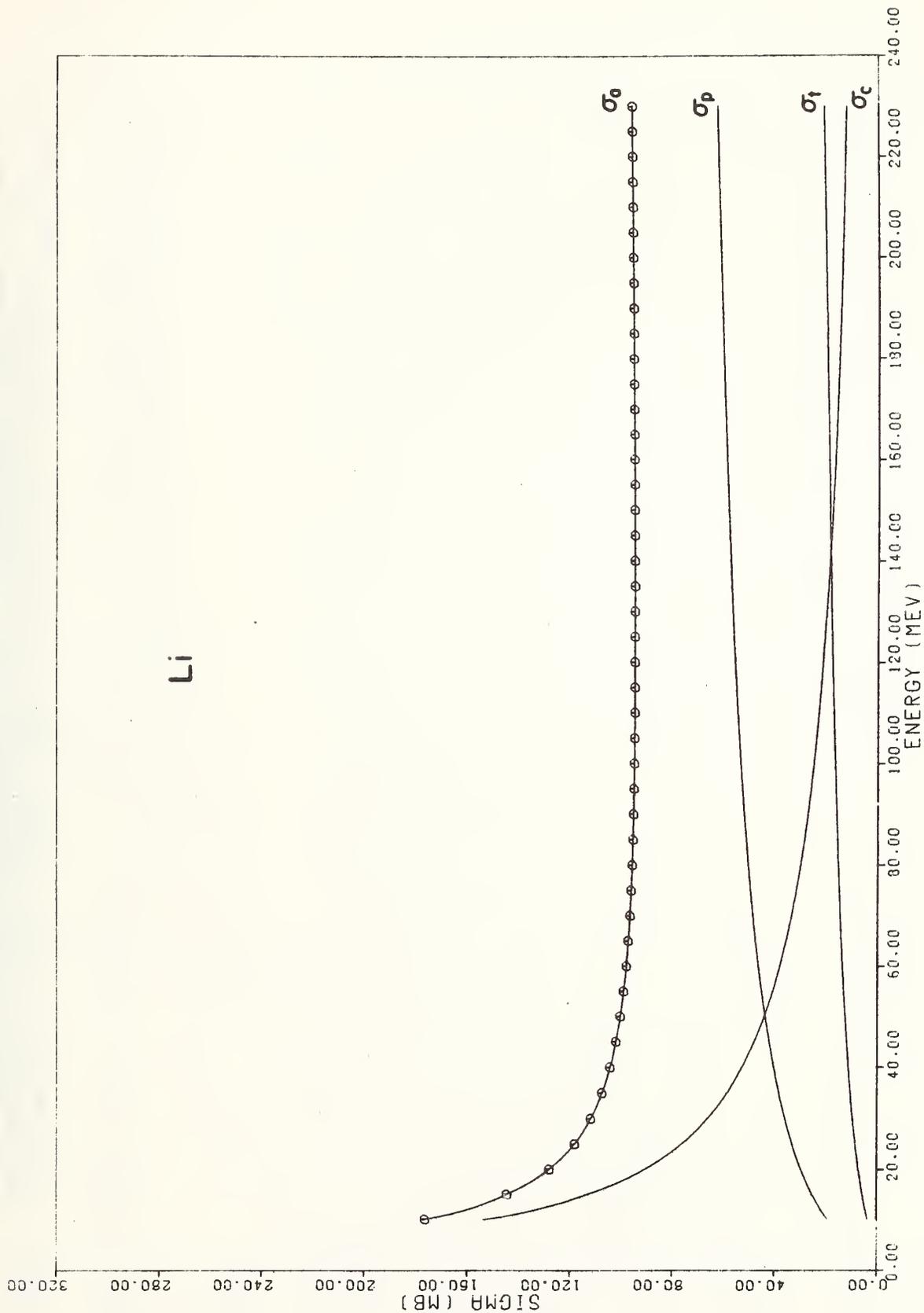


Figure 1 Calculated atomic cross sections,  $\sigma_a(k)$ , (circles) for Li. The partial cross sections for Compton effect,  $\sigma_c(k)$ , for pair production in the nuclear Coulomb field,  $\sigma_p(k)$  and for pair production in the electron field,  $\sigma_t(k)$ , are also given. The atomic photoelectric cross section,  $\sigma_{ph}(k)$ , is too small to be displayed in this scale.

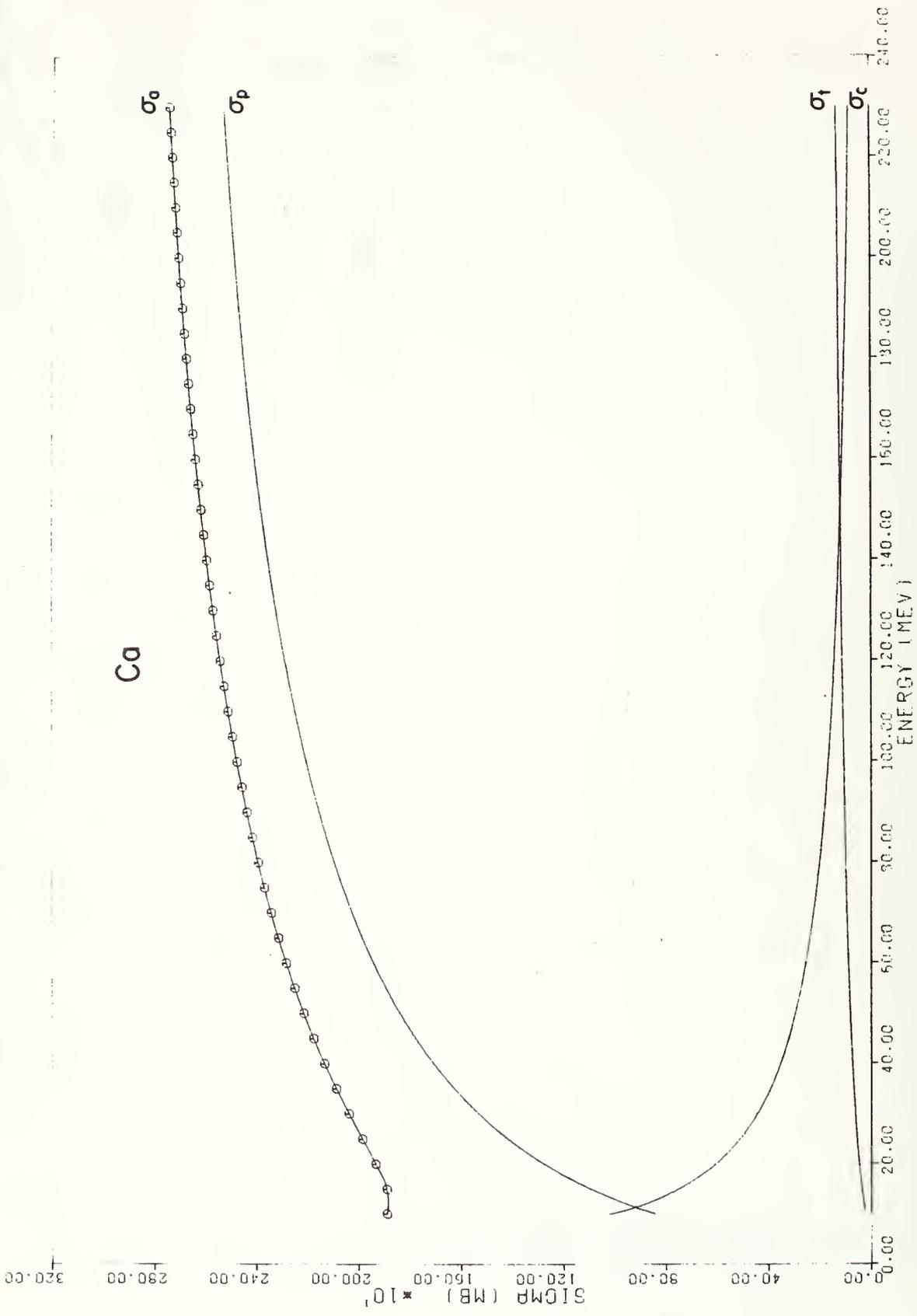


Figure 2 Calculated atomic cross sections,  $\sigma_a(k)$ , (circles) for Ca. The partial cross sections for Compton effect,  $\sigma_c(k)$ , for pair production in the nuclear Coulomb field,  $\sigma_p(k)$  and for pair production in the electron field,  $\sigma_r(k)$ , are also given. The atomic photoelectric cross section,  $\sigma_{ph}(k)$ , is too small to be displayed in this scale.

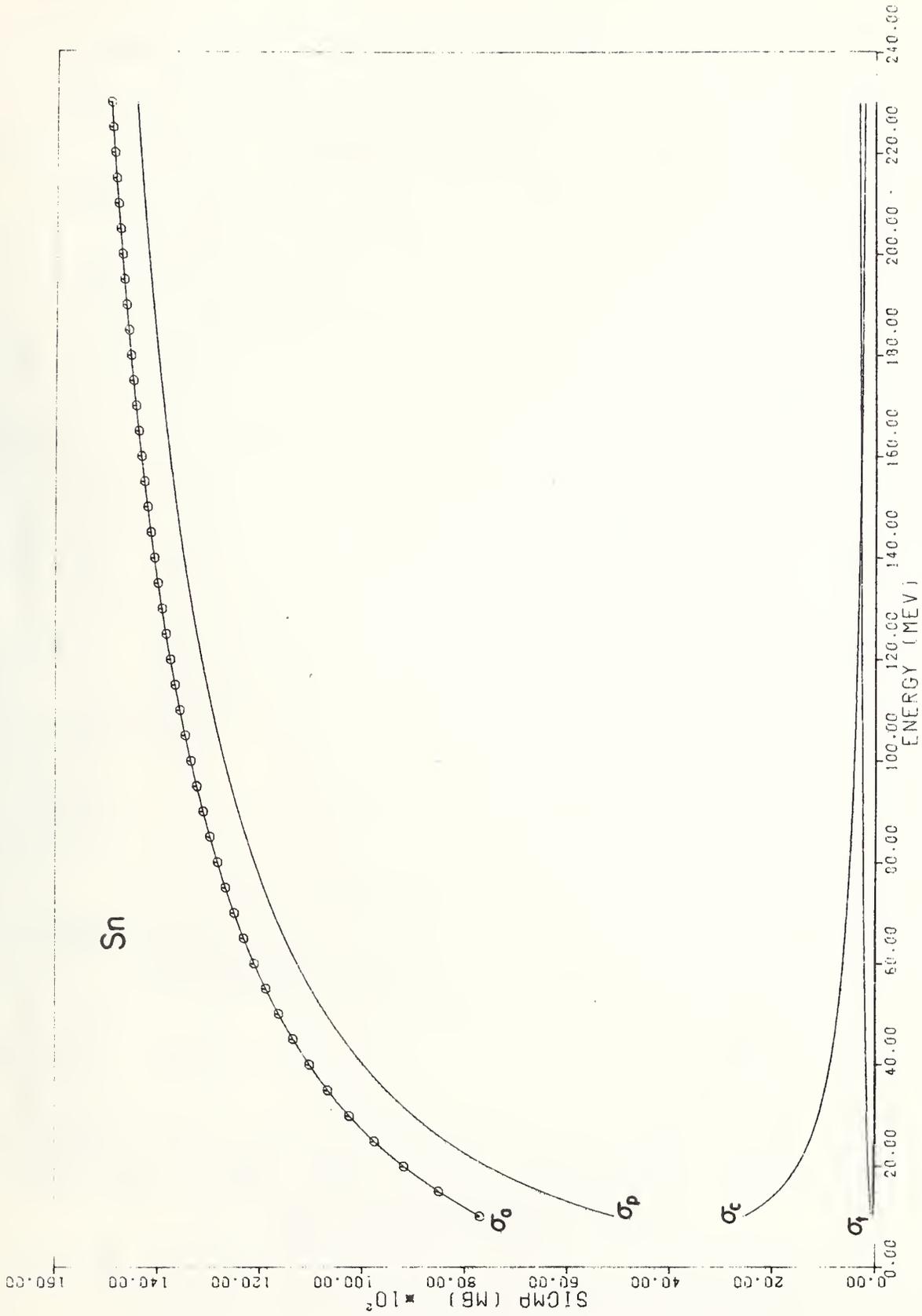


Figure 3 Calculated atomic cross sections,  $\sigma_a(k)$ , (circles) for Sn. The partial cross sections for Compton effect,  $\sigma_c(k)$ , for pair production in the nuclear Coulomb field,  $\sigma_p(k)$  and for pair production in the electron field,  $\sigma_t(k)$ , are also given. The atomic photoelectric cross section,  $\sigma_{ph}(k)$ , is too small to be displayed in this scale.

Pb

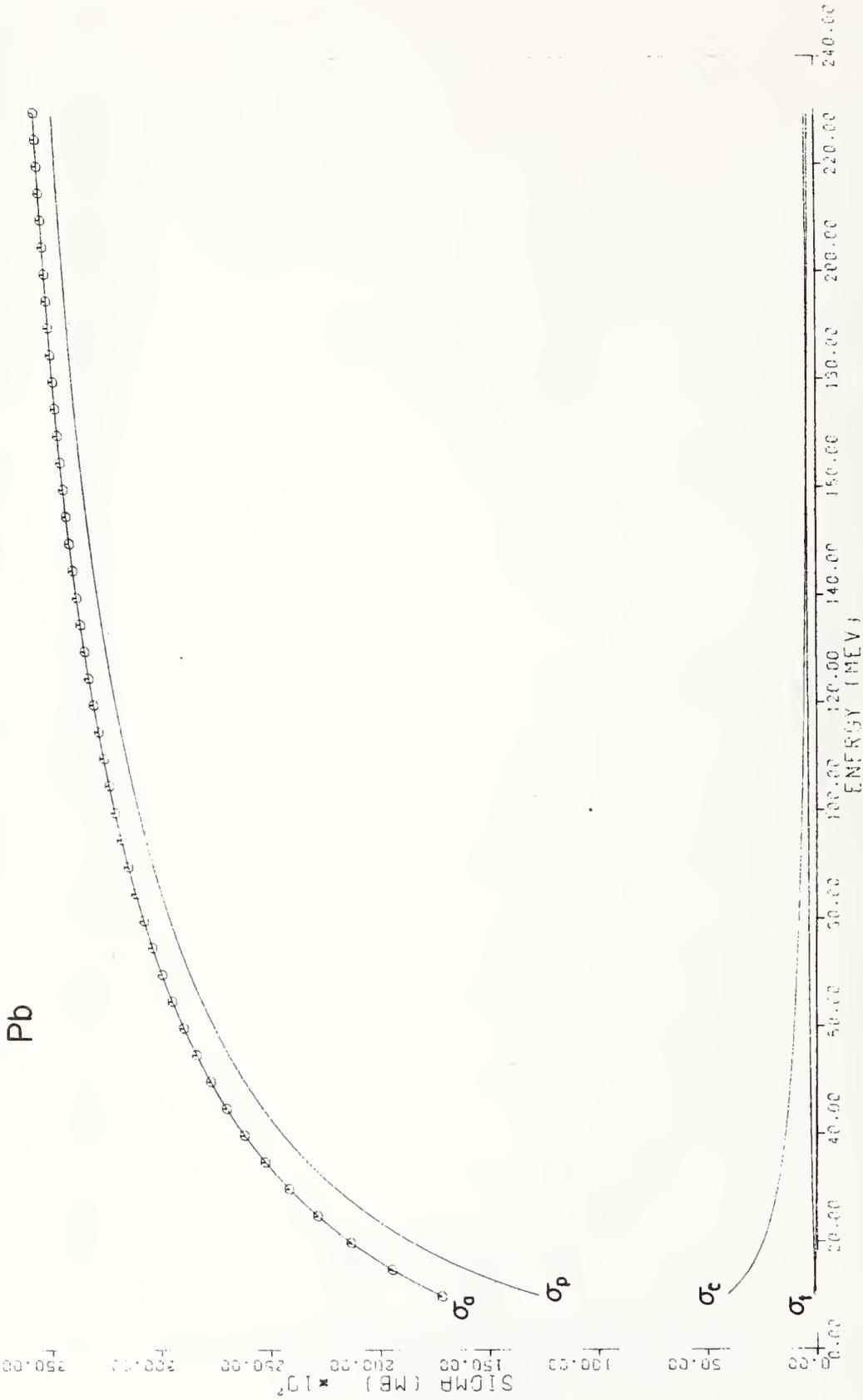


Figure 4 Calculated atomic cross sections,  $\sigma_a(k)$ , (circles) for Pb. The partial cross sections for Compton effect,  $\sigma_c(k)$ , for pair production in the nuclear Coulomb field,  $\sigma_p(k)$  and for pair production in the electron field,  $\sigma_t(k)$ , are also given. The atomic photoelectric cross section,  $\sigma_{ph}(k)$ , is too small to be displayed in this scale.

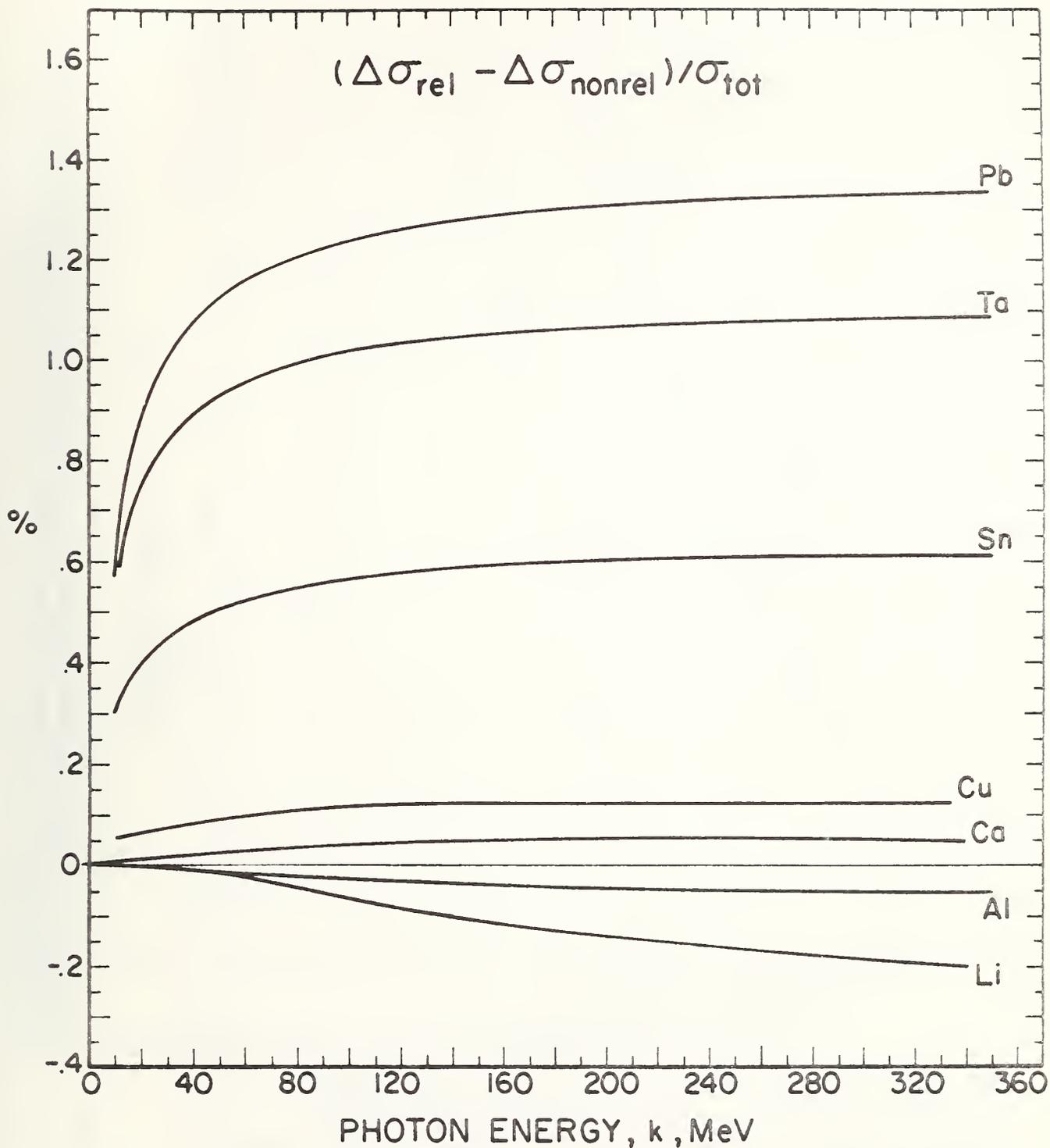


Figure 5 Difference between screening corrections calculated from relativistic and non-relativistic charge densities for the cross section for pair production in the field of a nucleus, in Born approximation, given as percentage of the total cross section,  $\sigma_{\text{tot}}$ , as a function of the photon energy,  $k$ , in MeV, for several elements.

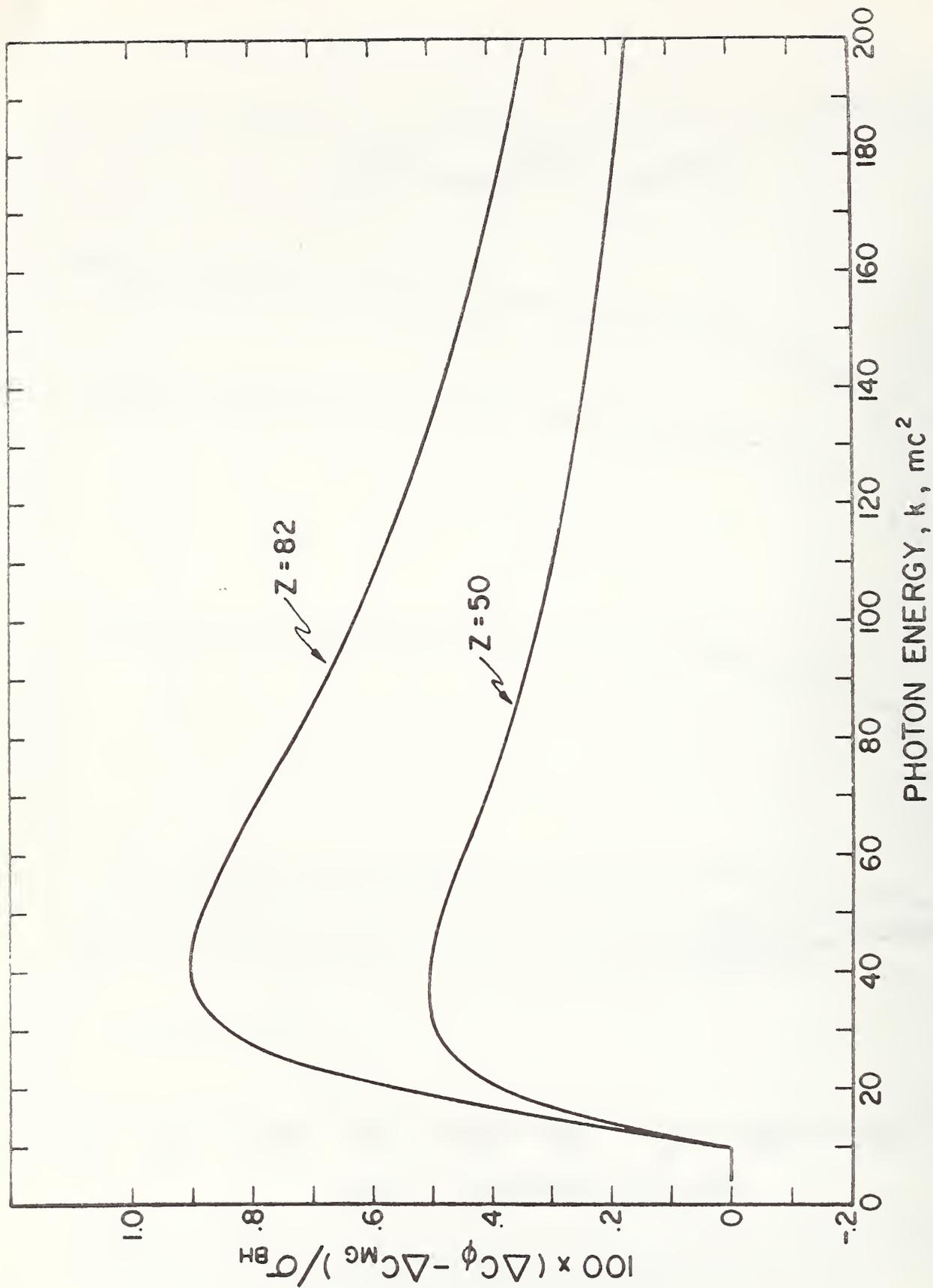


Figure 6 Ratio of the difference between the Coulomb correction calculated using our analytical expression ( $\Delta C_{MC}$ ) and that calculated by  $\phi_{verb\phi}$  using the expression in [15] ( $\Delta C_{\phi}$ ) to the Born approximation cross section ( $\sigma_{BH}$ ) as a function of photon energy,  $k$ , in  $mc^2$ , for  $Z = 50$  and  $Z = 82$ .

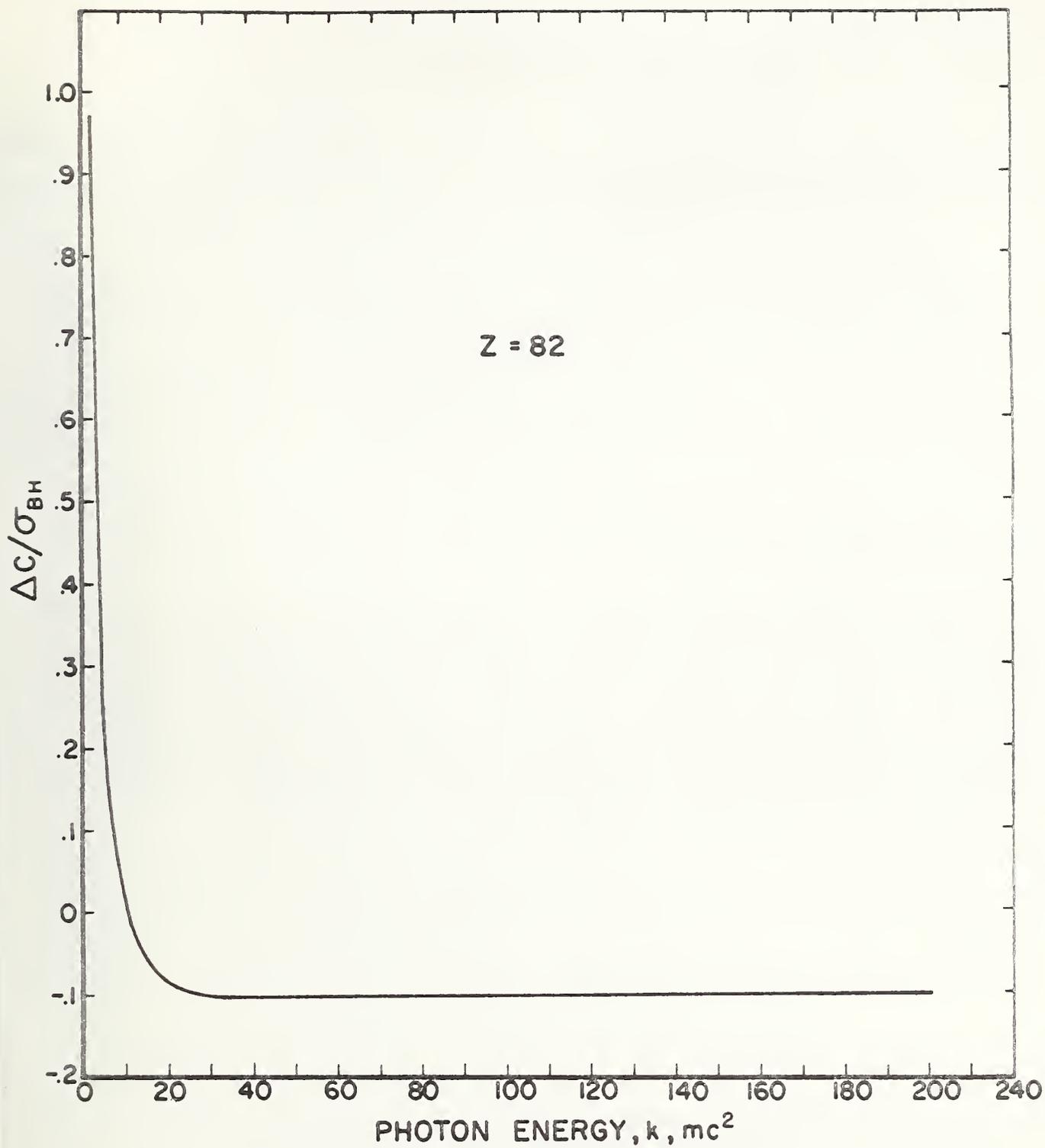


Figure 7 Ratio of the Coulomb correction to the unscreened Born approximation cross section as a function of the photon energy,  $k$ , in  $mc^2$ , for  $Z = 82$ .

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